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EFFICIENCY OF A THERMAL CURTAIN

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UDC 532.517.4

Gas curtains are widely used to protect surfaces washed by a high-enthalpy gas flow.

The main parameter describing the heat transfer under these conditions is the curtain efficiency

$$\theta = \frac{T_W^* - T_0}{T_{W_1} - T_0} = \frac{\delta_{T_1}^{**}}{\delta_{T_{ad}}^{**}},$$

where T_0 is the temperature of the unperturbed stream; T_W^* is the adiabatic wall temperature; T_{W_1} is the wall temperature at the curtain entrance point; $\delta_{T_{ad}}^{**}$ is the energy loss thickness on the adiabatic wall; and $\delta_{T_1}^{**}$ is the energy loss thickness at the curtain entrance point.

Several authors [1-3] have proposed analytical expressions to determine the efficiency of the thermal curtain; in [2, 3] these expressions were given for the limiting case $x \to \infty$.

However, in a number of cases of practical engineering importance the length of the protected surfaces is small, and there is therefore a need for more accurate determination of thermal curtain efficiency in the entrance section. An analytical expression for this case can be obtained from the following assumptions. It is well known [1, 2] that under these conditions the law of superposition of thermal fields is applicable, and one can therefore assume that a new thermal perturbation resulting from the effect of the wall being adiabatic will grow in the existing thermal boundary layer in the same way as the thermal boundary layer grows under the conditions of the preceding adiabatic section.

Figure 1 shows the temperature profile on an adiabatic wall (solid line). In order to show how a new thermal perturbation develops, i.e., the region with zero temperature gradient, it is convenient to represent the dimensionless temperature in the form

$$(T_{W_1} - T)/(T_{W_1} - T_0).$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 100-103, July-August, 1976. Original article submitted July 22, 1975.

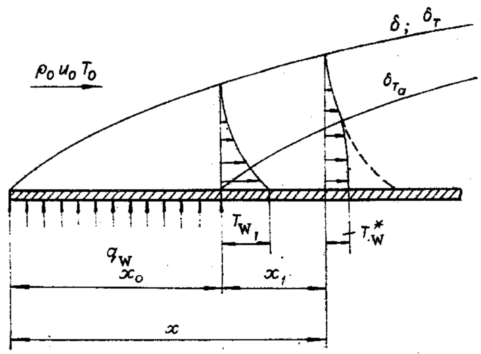

Fig. 1

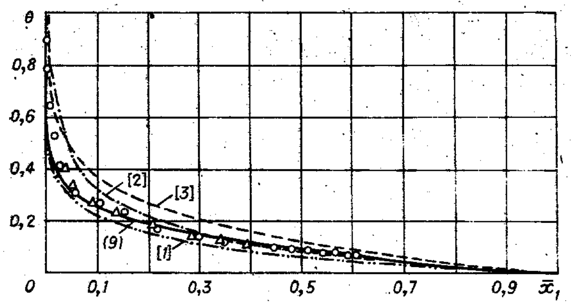

Fig. 2

This profile can then be divided into two regions: a region with zero temperature gradient, whose width increases with increase of x_1, and a region of unperturbed flow where the temperature profile retains its previous shape.

By definition the energy loss thickness in the adiabatic wall section is written in the form

$$\delta^{**}_{T_{\mathrm{ad}}} = \int_0^{\delta_T} \frac{W}{W_0}\left(1 - \frac{T^*_{\mathrm{w}} - T}{T^*_{\mathrm{w}} - T_0}\right) dy.$$

By means of simple transformations this expression can be reduced to the form

$$\delta^{**}_{T_{\mathrm{ad}}} = \frac{1}{\theta}\int_0^{\delta_T} \frac{W}{W_0}\left(1 - \frac{T_{\mathrm{w}_1} - T}{T_{\mathrm{w}_1} - T_0}\right) dy, \text{ where } \theta = \frac{T^*_{\mathrm{w}} - T_0}{T_{\mathrm{w}_1} - T_0}. \tag{1}$$

The integral in Eq. (1) can be divided into three parts (see Fig. 1):

$$\int_0^{\delta_T} \frac{W}{W_0}\left(1 - \frac{T_{\mathrm{w}_1} - T}{T_{\mathrm{w}_1} - T_0}\right) dy = \int_0^{\delta_T} \frac{W}{W_0} dy - \left[\int_0^{\delta_{T_a}} \frac{W}{W_0}\left(\frac{T_{\mathrm{w}_1} - T^*_{\mathrm{w}}}{T_{\mathrm{w}_1} - T_0}\right) dy + \int_{\delta_{T_a}}^{\delta_T} \frac{W}{W_0}\left(\frac{T_{\mathrm{w}_1} - T}{T_{\mathrm{w}_1} - T_0}\right) dy\right].$$

Thus, the expression for the energy loss thickness can be written in the form

$$\delta^{**}_{T_{\mathrm{ad}}} = \frac{\delta_T}{\theta}\left\{\int_0^1 \omega d\xi - \left[\int_0^{\delta_{T_a}/\delta_T} \omega(1-\theta)\, d\xi + \int_{\delta_{T_a}/\delta_T}^1 \omega\left(\frac{\Delta T}{\Delta T_1}\right) d\xi\right]\right\}, \tag{2}$$

$$\text{where } \omega = \frac{W}{W_0};\quad \frac{\Delta T}{\Delta T_1} = \frac{T_{\mathrm{w}_1} - T}{T_{\mathrm{w}_1} - T_0};\quad \xi = \frac{y}{\delta_T}.$$

Assuming that at the section x_1 the dynamic thickness δ and the thermal thickness δ_T of the layer are equal, and that the velocity and temperature profiles can be expressed as power series with the same exponent n, we can transform Eq. (2) as follows:

$$\frac{\theta\delta^{**}_{T_{\mathrm{ad}}}}{\delta_T} = \int_0^1 \xi^n d\xi - \left[\int_0^{\delta_{T_a}/\delta_T} \xi^n(1-\theta)\, d\xi + \int_{\delta_{T_a}/\delta_T}^1 \xi^{2n} d\xi\right],$$

and after integration it takes the form

$$\frac{\theta\delta^{**}_{T\text{ad}}}{\delta}=\left(\frac{1}{n+1}-\frac{1}{2n+1}\right)-\left(\frac{1-\theta}{n+1}\right)\left(\frac{\delta_{T\text{a}}}{\delta}\right)^{n+1}+\left(\frac{1}{2n+1}\right)\left(\frac{\delta_{T\text{a}}}{\delta}\right)^{2n+1} \tag{3}$$

By dividing the left and right sides of Eq. (3) by the quantity

$$\frac{\delta^{**}_{T_1}}{\delta_1}=\left(\frac{1}{n+1}-\frac{1}{2n+1}\right),$$

we obtain

$$\frac{\delta_1}{\delta}=1-(1-\theta)\left(\frac{2n+1}{n}\right)\left(\frac{\delta_{T\text{a}}}{\delta}\right)^{n+1}+\left(\frac{n+1}{n}\right)\left(\frac{\delta_{T\text{a}}}{\delta}\right)^{2n+1}.$$

Assuming that this thickness of the new thermal perturbation $\delta_{T\text{a}}$ varies with distance x_1 according to the same law as the thickness of a thermal layer on a flat plate with a preceding adiabatic section, and using the well-known dependence [2] $\text{Re}_\delta = A\ \text{Re}_x^{(n+1)/(3n+1)}$, we obtain

$$\theta=1-\frac{1-\bar{x}_0^{\frac{n+1}{3n+1}}+\frac{n+1}{n}\Pr^{-0.75(2n+1)}\bar{x}_1^{\frac{(n+1)(2n+1)}{3n+1}}}{\frac{2n+1}{n}\Pr^{-0.75(n+1)}\bar{x}_1^{\frac{(n+1)^2}{3n+1}}}.$$

For the case n = 1/6 the expression for the thermal curtain efficiency takes the form

$$\theta=1-\frac{1-\bar{x}_0^{7/9}+9.65\bar{x}_1^{56/54}}{10.57\bar{x}_1^{49/54}}. \tag{4}$$

Figure 2 shows results of curtain efficiency calculations using Eq. (4) and the previous formulas. For comparison Fig. 2 also shows experimental data of [1] and data of an experimental investigation of heat transfer in the entrance section of a curtain, obtained at the N. É. Bauman, Moscow Higher Technical School by Belov, Afanas'ev, and the present author. The relation suggested gives better agreement with experiment over the entire length of the protected surface.

The relations obtained in [2, 3] give overestimated values of curtain efficiency in the entrance section and give satisfactory agreement with experiment only from distances of $\bar{x}_1 > 0.3$ onward. The relation suggested by Seban [1] gives overestimated values of curtain efficiency over the entire length of the protected surface.

However, it should be noted that at very small distances from the curtain entrance point ($0 < \bar{x}_1 \leq 0.04$) calculations using Eq. (4) do not agree with experiment, since in this region a new thermal perturbation has not yet emerged from the viscous region of the turbulent boundary layer, and therefore, it is invalid to assume that the velocity intensity profiles can be described by power relations with a constant exponent.

The author thanks Professor A. I. Leont'ev for useful advice.

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